CSE-4303/CSE-5365 Computer Graphics   Fall 2024 Exam_02

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Prob #	**1**	**2**	**3**	**4**	**Total:**
Points	**16**	**28**	**28**	**28**	**100**

Time: 120 Minutes

Seat Number:

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Given a vector: $V = \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix}$

Rotate a vector around x axis until it lies in the xz plane	Rotate a vector around y axis until it lies in the yz plane	Rotate a vector around z axis until it lies in the yz plane
$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c}{\sqrt{b^2+c^2}} & \frac{-b}{\sqrt{b^2+c^2}} & 0 \\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$R_y = \begin{bmatrix} \frac{c}{\sqrt{a^2+c^2}} & 0 & \frac{-a}{\sqrt{a^2+c^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{a}{\sqrt{a^2+c^2}} & 0 & \frac{c}{\sqrt{a^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$R_z = \begin{bmatrix} \frac{b}{\sqrt{a^2+b^2}} & \frac{-a}{\sqrt{a^2+b^2}} & 0 & 0 \\ \frac{a}{\sqrt{a^2+b^2}} & \frac{b}{\sqrt{a^2+b^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$M_{Hermite} = \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \qquad M_{Bezier} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$R = 2N(N \cdot L) - L$$

$$t = -(\vec{OC} \cdot \vec{D}) \pm \sqrt{(\vec{OC} \cdot \vec{D})^2 - (\vec{OC} \cdot \vec{OC} - R^2)}$$

How to convert a general parallel view volume into canonical perspective volume

Step 1: Translate VRP to origin
Step 2: Rotate VPN around x until it lies in the xz plane with positive z
Step 3: Rotate VPN around y until it aligns with the positive z axis.
Step 4: Rotate VUP around z until it lies in the yz plane with positive y
Step 5: Shear DOP such that it aligns with vpn.
Step 6: Translate the lower corner of the view volume to the origin
Step 7: Scale such that the view volume becomes a unit cube

DOP= CW-PRP (CW: Center of Window on the View Plane)

How to convert a general perspective view volume into canonical perspective volume

Step 1: Translate VRP to origin
Step 2: Rotate VPN around x until it lies in the xz plane with positive z
Step 3: Rotate VPN around y until it aligns with the positive z axis.
Step 4: Rotate VUP around z until it lies in the yz plane with positive y
Step 5: Translate PRP (COP) to the origin
Step 6: Shear such that the center line of the view volume becomes the z axis
Step 7: Scale such that the sides of the view volume become 45 degrees
Step 8: Scale such that the view volume becomes the canonical perspective volume

Note: the two scale matrices in steps 7 and 8 can be combined into a single scale matrix.

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1. Given the plane P as **4x-3y+2z-12=0** and two points **A(3,2,5)** and **B(6,2,5).**

   a. Find the sequence of matrices to rotate plane P **90 degrees around line $\overrightarrow{AB}$**

   b. Show the equation of plane after the transformation

Show the sequence of matrices:

Calculate vector AB and translate point A to origin

$$\overrightarrow{AB} = B - A = \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ **(2 points)**

Matrices for Rotation

Rotate by 90 degrees around the x-axis. The rotation matrix is:

$$R_x(90^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ **(4 points)**

Translate back

Translate point A back to its original position. The inverse translation matrix is:

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ **(2 points)**

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The sequence of matrices :

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Show the equation of the plane P after rotation:

The original plane equation is:

$$4x - 3y + 2z - 12 = 0$$

The normal vector to the plane is:

$$N = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$$

After rotation, the normal vector is transformed using the rotation matrix:

$$N' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ -3 \\ 1 \end{bmatrix}$$

The new plane equation is:

4x - 2y - 3z + d = 0 **(2 points)**

To find d, find a point on the original plane (before transformations) such as:

$$P = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$$ **(2 points)**

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Transform this point, P, using the three matrices

$$P' = T^{-1} \cdot R_x(90^\circ) \cdot T \cdot P = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$ **(2 points)**

Substitute the transformed point into the new plane equation.

The equation of the transformed plane:

4x - 2y - 3z + 11 = 0 **(2 points)**

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Problem 1 Continued:

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2. The geometry vector of a cubic-linear surface in the u direction is defined by Bezier and in the v direction is defined as $\left[\boldsymbol{p}_0, \frac{\mathbf{d}\boldsymbol{p}_0}{\mathbf{dv}}\right]$

Find the geometry matrix [G] for this surface.

Note: All elements should be specified explicitly as $\boldsymbol{p}_{mn}$ or derivatives of it such as $\frac{\mathbf{d}\boldsymbol{p}_{mn}}{\mathbf{du}}$ or $\frac{\mathbf{d}^2\boldsymbol{p}_{mn}}{\mathbf{dudv}}$. **Do not** use implicit forms such as p_1, p_2, p_3, p_4.

$$G_{BL} = \begin{bmatrix} p_{00} & \frac{dp_{00}}{dv} \\ p_{00} + \frac{1}{3}\frac{dp_{00}}{du} & \frac{dp_{00}}{dv} + \frac{1}{3}\frac{d^2 p_{00}}{dudv} \\ p_{10} - \frac{1}{3}\frac{dp_{10}}{du} & \frac{dp_{10}}{dv} - \frac{1}{3}\frac{d^2 p_{10}}{dudv} \\ p_{10} & \frac{dp_{10}}{dv} \end{bmatrix}$$

(16 points)

Given the equation of the above parametric surface

$$\boldsymbol{S}(\boldsymbol{u},\boldsymbol{v}) = \mathbf{4}\boldsymbol{u}^3\boldsymbol{v} + \mathbf{9}\boldsymbol{u}\boldsymbol{v} - \mathbf{6}\boldsymbol{u} + \mathbf{10}$$

Find the numerical values of the geometry matrix [G] for this surface.

You must show the numerical values to receive credit.

$$G_{BL} = \begin{bmatrix} 10 & 0 \\ 8 & 3 \\ 6 & 6 \\ 4 & 13 \end{bmatrix}$$

(12 points)

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Problem 2 Continued:

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3. Given the triangle ABC in a three dimensional right-handed coordinate system,

A=(2,2,0), B=(0,3,0), C=(0,0,4)

The light source with an intensity of I=1000 is located at **(7,6,8)** and the viewer (eye) is located at **(10,12,8)** and ; **K_d=0.1 ; K_s=0.5 ; n=2**

- Ignore fatt

a. Find the diffuse intensity at point A

$$\overrightarrow{AB} = B - A = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$ **(1 point)**

$$\overrightarrow{AC} = C - A = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 4 \end{bmatrix}$$ **(1 point)**

Normal to the plane of the triangle is the cross product of AB and AC

$$\overrightarrow{N} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} X \begin{bmatrix} -2 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 6 \end{bmatrix}$$ **(2 points)**

Normalized

$$\overrightarrow{N} = \begin{bmatrix} 0.37139068 \\ 0.74278135 \\ 0.55708601 \end{bmatrix}$$ **(2 points)**

Light vector:

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$$\vec{L} = L - A = \begin{bmatrix} 7 \\ 6 \\ 8 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 8 \end{bmatrix}$$ **(2 points)**

Normalize L

$$\vec{L} = \begin{bmatrix} 0.48795004 \\ 0.39036003 \\ 0.78072006 \end{bmatrix}$$ **(2 points)**

$$I_d = K_d \cdot I \cdot (\vec{L} \cdot \vec{N}) = 90.610$$

$$(\vec{L} \cdot \vec{N}) = 90.610$$

$$I_d = K_d \cdot I \cdot (\vec{L} \cdot \vec{N}) = 90.610$$ **(2 points)**

b. Find the specular intensity at point A

Find reflection vector

$$\vec{R} = 2(\vec{L} \cdot \vec{N})\vec{N} - \vec{L} = 2*90.610* \begin{bmatrix} 0.37139068 \\ 0.74278135 \\ 0.55708601 \end{bmatrix} - \begin{bmatrix} 0.48795004 \\ 0.39036003 \\ 0.78072006 \end{bmatrix} = \begin{bmatrix} 0.1850845 \\ 0.95570904 \\ 0.22883174 \end{bmatrix}$$ **(4 points)**

Normalize R

$$\vec{R} = \begin{bmatrix} 0.1850845 \\ 0.95570904 \\ 0.22883174 \end{bmatrix}$$ **(2 points)**

Calculate V

$$\vec{V} = V - A = \begin{bmatrix} 10 \\ 12 \\ 8 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 8 \end{bmatrix}$$ **(4 points)**

Normalize

$$\vec{V} = \begin{bmatrix} 0.52981294 \\ 0.66226618 \\ 0.52981294 \end{bmatrix}$$ **(2 points)**

$$(\vec{R} \cdot \vec{V}) = \begin{bmatrix} 0.1850845 \\ 0.95570904 \\ 0.22883174 \end{bmatrix} \cdot \begin{bmatrix} 0.52981294 \\ 0.66226618 \\ 0.52981294 \end{bmatrix} = 0.85223$$ **(2 points)**

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$I_s = K_s \cdot I \cdot (\vec{R} \cdot \vec{V})^n = 0.5*1000*(0.85223)^2 = 363.14964$ **(2 points)**

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Problem 3 Continued:

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4. The viewing parameters for a perspective projection are given as

VRP(WC)=**(0,0,0)** VPN(WC)=**(0, 2,0)**

VUP(WC)=**(2,1,-2)** PRP (VRC)=**(-2,6,-5)**

u_{min} (VRC) = **12** u_{max} (VRC) = **14**

v_{min} (VRC) = **-14** v_{max} (VRC) = **11**

n_{min} (VRC) = **-2** n_{max} (VRC) = **-1**

Find the sequence of transformations which will transform this viewing volume into a standard perspective view volume which is bounded by the planes: x=z ; x=-z ; y=z ; y=-z ; z=1 ; z=zmin

a. Find the **sequence of matrices**
b. Find the **zmin** after all transformations are done.

Note: all numerical values should be calculated with **3 digits** of accuracy after the decimal point

Use the Excel sheet to calculate correct values

Matrix #2: Rx (2 points)

Matrix #1: Translate

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #4: Rz (2 points)

Matrix #3: Ry (2 points)

Matrix #6 Shear (6 points)

Matrix #5: Translate (2 points)

Matrix #8 Scale (6 points)

Matrix #7 Scale (6 points)

Zmin= (2 points)

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Problem 4 Continued: